Boundary-Layer Flows of Non-Newtonian Power Law Fluids

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Abstract: Non-Newtonian fluids are something which require lots of study and research on it flow pattern over the surfaces. This paper highlights the laminar flow of non-Newtonian fluids which obeys the power-law model past horizontal flat plate is presented. The main problem considered is that of predicting the drag and rate of heat transfer from an isothermal surface to the fluid. This paper proposes the detailed study of the flow past a horizontal flat plate in which the shear stress coefficient is determined exactly by numerical solution of the governing equation and approximately by a momentum integral method, analogous to the one developed by Pohlhausen for Newtonian fluids. Results obtained are then compared between the exact and approximate methods.The rate of heat transfer for flow past an isothermal flat plate is determined by numerical solution of energy equation, approximately by an integral method. Numerical solution of momentum and energy equations is obtained by finite difference procedure by using central difference scheme. Results obtained for heat transfer rate by numerical method, integral method and Lighthills approximation are compared.

1 INTRODUCTION

Due to the importance of applications of non-Newtonian fluids in industries, processing molten plastics, polymers, pulps, slurries, emulsion, etc. Considerable efforts have been conducted to understand the behavior of non-Newtonian fluids. These fluids do not obey the Newtonian postulate that the stress tensor is directly proportional to the deformation tensor.

In non-Newtonian fluid there is no definite relationship between stress tensor and the deformation tensor valid for all fluids which lead to the difficulty in the theoretical study of non-Newtonian fluid mechanical phenomenon and in the correct interpretation of experimental results. This means that except for simple cases, a generalized form of the Navier-Stokes equations, obeyed by all fluids in motion, cannot be written down.

There are number of models suggested for the non-Newtonian fluids but Power-law model is a most popular and simplest model. Power-law model is adequate for many non-Newtonian fluids and it is widely used.

The momentum equation is strongly non-linear and hence similarity solution for the heat transfer is difficult to obtain. The similarity can be obtained only for infinite Prandtl numbers i.e. when thermal boundary layer thickness is much smaller than the velocity boundary layer.

Acrivos, Shah, Petersen [1] were the first to study momentum and heat transfer in laminar boundary-layer flows of non-Newtonian fluids which obeys power law model past external surfaces. They considered how to predict the drag and rate of heat transfer from an isothermal surface to the fluid. Local heat transfer rates were estimated using Lighthills approximate formula, on the assumption that Prandtl number $\rightarrow \infty$, i.e. the thermal boundary-layer is much thinner than the shear layer.

Ames and Lee [2] have reviewed the existing important similar solutions available in literature, for boundary-layer flows of non-Newtonian fluids. They have applied transformation group method to obtain similarity variables and equations for various types of flows, e.g. Falkener-Skan flows and Goldstein flows. The numerical solutions of the forced convection of power-law fluids for right angle wedge with an isothermal surface have been presented.

A. K. Kulkarni, H. Jacobs, J. Hwang [3] obtained a similarity solution for a natural convection flow on a heated isothermal wall suspended in a quiescent, thermally stratified atmosphere. They also represented the case for an isothermal plate in a linearly stably stratified atmosphere. R. henkes, C. Hoogendoorn [4] determined numerically similarity solution of the laminar natural convection boundary layer equations for air for a fixed wall and variable environment temperature.A theoretical analysis of natural convection to power-law fluids from a heated vertical plate in a stratified environment was first explained by Lee, Gorla, and Pop [5].

Kumari, Pop [6] did the theoretical analysis of laminar free convection flow over a vertical isothermal wavy surface in non-Newtonian power law fluids. An implicit finite difference method known as Keller-Box method used to solve the boundarylayer equations. A sinusoidal surface used for analysis. They have shown that the local Nusselt number varies periodically along the wavy surface.The transient convection heat transfer in a power law fluid is of major interest and a numerical solution of the appropriate unsteady boundary layer equations presented by Haq, Kleinstreuer [7].Ece and Buyuk [8] presented the similarity solution for power law fluids from a vertical plate under mixed thermal boundary conditions.. M. J. Huang, J. Huang, C. K. Chen [24] studied the effects of Prandtl number on free convection heat transfer from a vertical plate to a non-Newtonian fluid. The analysis includes the inertia force in the momentum equation with a finite Prandtl number.

A theoretical analysis for forced convection heat transfer from external surfaces immersed in non-Newtonian fluids of power law model is done by A. Nakayama, A. V. Shenoy and H. Koyama [9]. F. N. Lin and S. Y. Chern [10] presented a solution for the two-dimensional and axis-symmetrical laminar boundary-layer momentum equation of power-law nonNewtonian fluid. They used Merk-Chao series solution method. H. Pascal [23] presented similarity solutions to some unsteady flows of nonNewtonian fluids of power law behaviour. He addressed non-linear effects in some unsteady flows. From the solutions the conditions for the existence of travelling wave characteristics are found for the velocity, shear stress and pressure distributions. H. Aderson, T. Toften [26] explained a numerical solution for laminar boundary layer equations for power law fluid using implicit finite difference keller box scheme.R. Mahalingam [28] calculated local rates of heat transfer coefficient for non-Newtonian power law Pseudoplastic liquids in laminar flow in circular conduits. A. V. Shenoy[29] described criteria for transition to turbulence during natural convective heat transfer from a flat vertical plate to power law fluid.

2 NON-NEWTONIAN FLUIDS

Non Newtonian fluids are those fluids for which the flow curve(τ versus du/dy) is not linear through the origin at a given temperature and pressure. These materials are commonly divided into three broad groups.

- Time-independent fluids,
- Time-dependent fluids,
- Viscoelastic fluids.

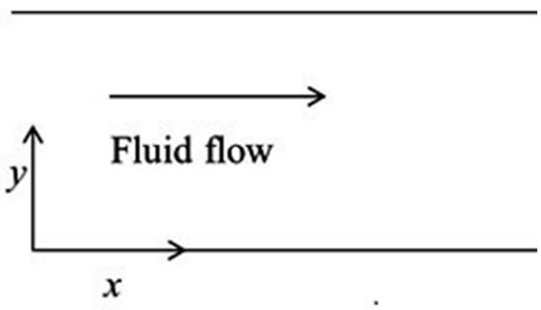

Fig. Non-Newtonian flow between two parallel plates

3 ANALYSIS OF FLOW BEHAVIOUR

Consider the laminar flow of a non-Newtonian fluid past the arbitrary two-dimensional surface.

Assumptions:

i. Constant property fluid.
ii. Dissipation is neglected.
iii. Two dimensional and steady.
iv. Gradients in the normal direction are much larger than the gradients in the transverse, or x, direction.

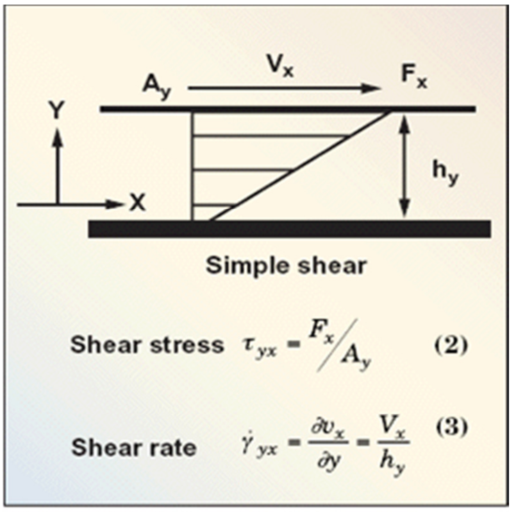

FIGURE 1. In this diagram, with fluid between two planes, as the top plane moves, it stresses the fluid and defines the shear rate

Power law or Ostwald De Waele model is the most generalized model for non-Newtonian fluids.

$$\tau_{yx} = -m\left|\left(\frac{dv_x}{dy}\right)\right|^{n-1}\frac{dv_x}{dy}$$

Here, apparent viscosity η is defined as,

$$\eta = m\left|\left(\frac{dv_x}{dy}\right)\right|^{n-1}$$

Where *m* and *n* are the two parameters.

If $n = 1$ then η = m
Where *m* is similar to the viscosity of the fluid and model shows the Newtonian behaviour.

If $n>1$, then η increases with increasing shear rate and the model shows the Dilatant behaviour.

If $n<1$, then η decreases with increasing shear rate and the model shows the Pseudo-plastic behaviour.

Eyring model is a two-parameter model. The equation of Eyring model is as follow

$$\sinh\left(\frac{\tau_{yx}}{A}\right) = -\frac{1}{B}\frac{dv_x}{dy} \qquad (20.8)$$

Where A, B are the two parameters. In Eyring model, if $\tau_{xy} \rightarrow 0$ which means very low shear forces, we have

$$sinh\left\{\frac{\tau_{yx}}{A}\right\} \rightarrow \frac{\tau_{yx}}{A} \qquad (20.9)$$

Therefore, as $\tau_{yx} \rightarrow 0$, the model shows Newtonian behaviour

$$\tau_{yx} = -\frac{A}{B}\frac{dv_x}{dy} \qquad (20.10)$$

Here, viscosity = $\left(\frac{A}{B}\right)$ If τ_{yx} is very large, the model shows Non-Newtonian behaviour as shown Fig. 20.3

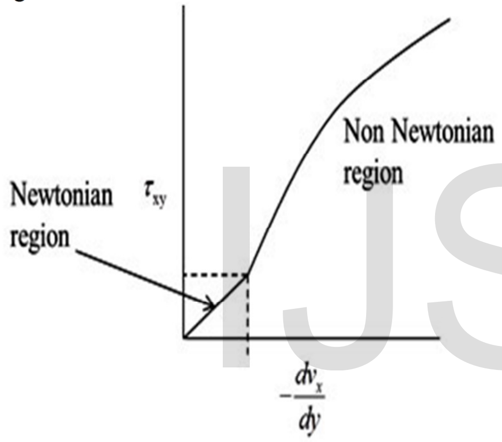

Fig. Shear stress vs. shear strain diagram for Eyring model

Therefore, Eyring model may be used for a fluid which shows Newtonian behaviour at low shear rates and non- Newtonian behaviour at high shear rates. 3. Ellis model Ellis model is a three-parameter model. The equation of this model is as follows

$$-\frac{dv_x}{dy} = \left\{\varphi_o + \varphi_1 \left|\tau_{yx}\right|^{\alpha-1}\right\}\tau_{yx}$$

Here, φ_0, φ_1 and α are the three parameters .

Here, we consider some special cases, 1. If $\varphi_1 = 0$ then Equation (20.11) reduce to

$$\frac{dv_x}{dy} = -\varphi_0 \tau_{yx}$$

or

$$\tau_{yx} = -\frac{1}{\varphi_0}\frac{dv_x}{dy}$$

Which is same as Newton's law of viscosity with $\left(\frac{1}{\varphi_0}\right)$ as the viscosity of the fluid.

2. If $\varphi_0 = 0$, then

$$-\frac{dv_x}{dy} = -\varphi_1 \left|\tau_{yx}\right|^{\alpha-1}\tau_{yx} \qquad (20.14)$$

Which is similar to a Power law model 3. If $\alpha > 1$ and τ_{yx} is small then the second term is approximately zero and equation reduces to

$$\tau_{yx} = -\frac{1}{\varphi_0}\frac{dv_x}{dy}$$

Which is similar to Newton's law of viscosity. 4. If $\alpha < 1$ and τ_{yx} is very large, then again, second term is negligible and we have

$$\tau_{yx} = -\frac{1}{\varphi_0}\frac{dv_x}{dy}$$

Which again shows Newtonian behaviour. Therefore, Ellis model may be used for fluids which show Newtonian behaviour at very low and very high shear stresses, but non-Newtonian behaviour at intermediate value of shear stresses.

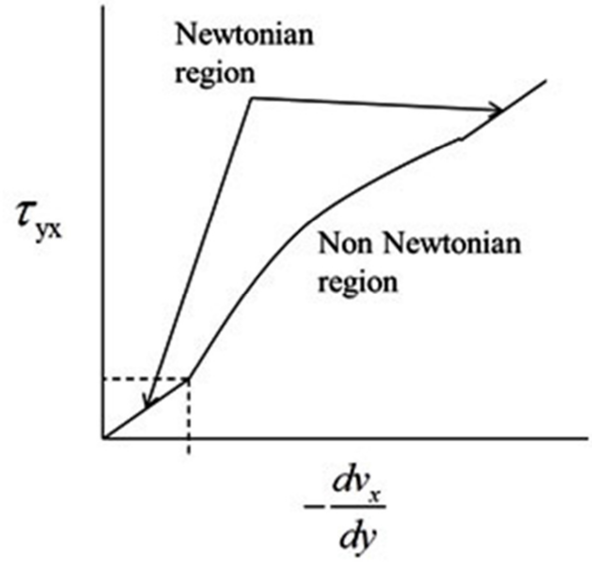

Fig. Shear stress vs. shear strain diagram for Ellis model

This type of behaviour may be shown by some polymer melts 4. Reiner Philipp off model this is also a three-parameter model. The equation of Reiner Philipp off model is as follows,

$$-\frac{dv_x}{dy} = \left[\frac{1}{\mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + \left(\frac{\tau_{yx}}{\tau_s}\right)^2}} \right] \tau_{yx} \qquad (20.16)$$

Where, μ_0 , μ_∞ and τ_s are the three parameters.

In Reiner Philipp off model, if τ_{yx} is very large, the equation reduces to,

$$-\frac{dv_x}{dy} = \frac{1}{\mu_\infty} \tau_{yx}$$

or

$$\tau_{yx} = -\mu_\infty \frac{dv_x}{dy}$$

Which is same as the Newton's law of viscosity, If τ_{yx} is very small then equation reduces to

$$-\frac{dv_x}{dy} = \frac{1}{\mu_0} \tau_{yx}$$

Or

$$\tau_{yx} = -\mu_0 \frac{dv_x}{dy}$$

Which is also same as the Newton's law of viscosity. Therefore, Reiner Philipp off model may be used for a fluid which shows Newtonian behaviour at very low and very high shear stresses but non-Newtonian behaviour for intermediate values of shear stress. Here, μ_0 and μ_∞ represent the viscosity of fluid at very low and very high shear stress conditions respectively. 5. Bingham Fluid model Bingham fluid is special type of fluid which require a critical shear stress to start the flow.The equation of Bingham fluid model are given below

$$\tau_{yx} = -\left[\mu + \frac{\tau_0}{\left|\frac{dv_x}{dy}\right|} \right] \frac{dv_x}{dy}$$

if

$$|\tau_{yx}| > \tau_0$$

$$\frac{dv_x}{dy} = 0 \quad \text{if } |\tau_{yx}| \le \tau_0$$

or

$$\eta = 0$$

A typical shear stress vs. shear rate diagram for a Binghum model is shown below

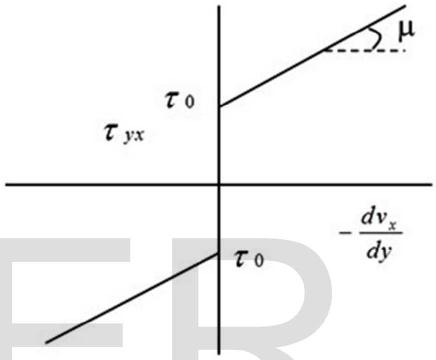

Fig. Shear stress vs. shear strain diagram for Bingham model

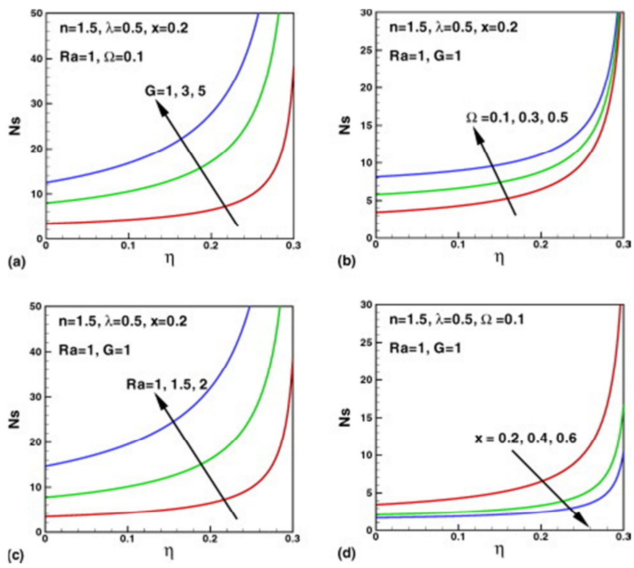

Fig. Variation of entropy

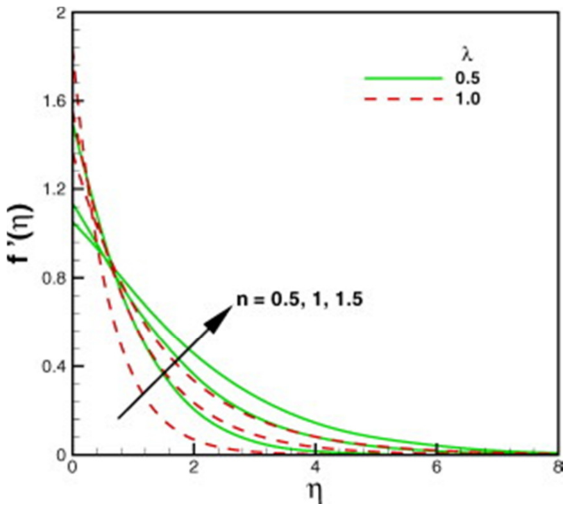

Fig. Velocity profiles showing effects of λ and different power-law fluids

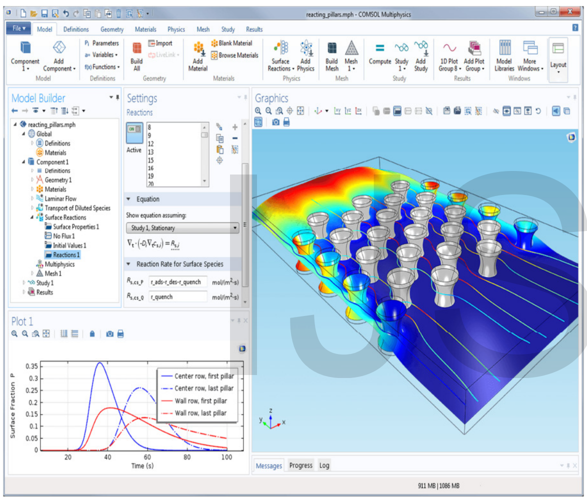

Fig. The shear rate, dynamic viscosity and volumetric flow of a polystyrene solution

4 CONCLUSION

The following conclusions can be drawn from the analysis-

- For n < 2 boundary layer type of flow can be obtained if ∞ is large and therefore the Reynold number is made sufficiently large.
- For n > 2 laminar boundary layer flows are not of much practical interest because their range of validity appears to be limited.
- For Pseudoplastic fluids as n increases velocity gradient increases while for dilatant fluids as value of n decreases velocity gradient decreases.
- Shear force decreases with increase in value of n for Pseudoplastic as well as dilatants fluids.
- Rate of heat transfer calculated by Integral method shows some discrepancies especially in the region of small flow behaviour index (n < 0.5).
- Local Nusselt number increases for Pseudoplastic fluids and decreases for Dilatant fluids from near the leading edge to the downstream position for same value of Prandtl number while for Newtonian fluids it remains constant.
- As Prandtl number increases rate of heat transfer increases for Pseudoplastic fluids, Newtonian fluids and Dilatant fluids.
- Asymptotic method holds good for large Prandtl number i.e. when thermal boundary layer is much thinner than the shear layer.

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